Spin chains as quantum channels

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1 Introduction

The task of finding a reliable solution to quantum communication and computation, based on real solid state devices, requires inputs from Quantum Information and Condensed Matter Physics. To achieve large-scale quantum computing, one must be able to link several tiny quantum processing devices. An interesting proposal is to use the quantum degrees of freedom of spin channels as qubits in order to establish quantum state transfer [1] or teleportation protocols [2]. Recent numerical simulations by Venuti and collaborators [3] showed that:

• the ground state of several spin chains has the ability to entangle two spins very far apart;
• this does not require external fields or complicated measurement schemes [2].

This strengthened the idea of using spin chains as quantum channels (see Figure 1) and led us to ask how general is long distance entanglement (LDE) mediated by spin chains.

Quantum frustration as the motivation for the weakly coupling regime

In systems with short range interactions, entanglement between two particles usually decays quickly with distance between them. Thus, entanglement between distant probes cannot arise from the entanglement among the spins with which they interact, but rather from an effective long-range interaction induced by the spin chain. Since we are interested in weakly interacting probes \((|i⟩ = |Jp⟩, \langle i|)\), we may compute the effective interaction by making use of the canonical transformation formalism.

The effective couplings induced by the spin chain

A system consisting of a gapped spin chain and two additional weakly coupled probes has a low energy sector separated by a gap \((\Delta)\) from the excited states. The effective Hamiltonian of the low energy sector can be calculated using the canonical transformation formalism:

\[
H_{\text{eff}} = H_0 - \frac{1}{2} [U, V] + O \left( \frac{1}{r^2} \right)
\]

The effective Hamiltonian parameters can be expressed quite generally in terms of spin chain correlation functions. This approach has the following advantages:

• it provides a theoretical framework to understand the mechanisms behind LDE;
• proves LDE analytically and clarifies the conditions for its existence in a broad class of spin chains.

2 The Heisenberg model

The low energy physics of many antiferromagnetic insulators is well described by the antiferromagnetic (AF) Heisenberg model:

\[
H = -\sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j
\]

The finite AF Heisenberg model: The finite chain is gapped and it has a finite \(\xi_0\). It can be computed using the conformal invariance of the critical chain.

The dynamical correlations in the Heisenberg model: The dominant long distance correlations in the AF Heisenberg ground state oscillate with a \(\pi\) phase change between neighbor spins.

General bosonization results [4] show that the Matsubara Green function of the staggered magnetization, \(M^A_{\pi/2}(t)\), for the infinite chain, reads \(\langle \vec{S}_j(0) \vec{S}_j(t) \rangle = A \sin(\omega_t)\), where \(A\) is an amplitude and \(\omega_t\) the Fermi velocity of the spin excitations. This implies that the infinite chain has a divergent \(\xi_0\), a direct consequence of a zero gap and a signal of the critical nature of the spin chain at \(t = 0\).

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SUSCEPTIBILITY FROM CONFORMAL MAPPING: The critical nature of the infinite chain implies that \(N\)-point correlation functions in different geometries are related by conformal transformations. The mapping of the infinite chain to the finite chain is achieved by a conformal transformation \((r, t) \rightarrow (w, \tau)\). Our perturbative approach cannot be applied un-

3 Other models

Analysis of the Affleck-Kennedy-Lieb-Tasaki (AKLT) biquadratic spin-1 model was made using a single-mode approximation to calculate time dependent correlations. We found results similar to those of the Heisenberg chain [5].

Our analysis suggests a very general physical mechanism behind LDE, namely dominant antiferromagnetic correlations between chain.

4 Scattering of probes

What happens when probes initially prepared in some separable state such as \(\{|\phi(0)⟩ = \{|+\rangle, -\rangle\}\) are scattered by interaction with the spin chain?

Since the low energy sector of spin chain and probes is well described by \(H_{\text{eff}}\), the latter will determine the time evolution by \(\phi(t) = \exp[ -i H_{\text{eff}} t] \phi(0)\). Using \(\langle \phi(0)| = 1 + 2P_{\phi}\) or \(\langle \phi(0)| = 1 - 2P_{\phi}\) for any \(\phi\), we have

\[
\langle \phi(0)| = \cos(2\chi_{\tau}(\phi)) \langle +\rangle - \sin(2\chi_{\tau}(\phi)) \langle -\rangle
\]

This equation describes entanglement oscillations and a maximal entangled pair is obtained for \(\tau = \pi/(\Delta_{\text{crit}})\). A similar scheme can be used to transfer the state of a third party to a distant probe.

5 Towards a multipartite entangler

Our method can also be used to study the interaction of \(N\) weakly interacting probes. In this case the effective Hamiltonian reads:

\[
H_{\text{eff}} = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j
\]

In general the eigenstates of the above Hamiltonian displays multipartite entanglement between spins and rich physics deserving a more detailed study.

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7 References