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The fluid dynamics of foams

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Abstract
Liquid foam is an example of soft matter (or a complex fluid) with a very well-defined structure, first clearly described by Joseph Plateau in the 19th century. Current research addresses many aspects of the fluid dynamics of this system. How is liquid transported through it in response to a pressure gradient or gravity? How does it respond to stress, particularly above the yield stress? What is the nature of the local fluid flow in the Plateau borders and their junctions? Simple first-order answers to many such questions exist but ongoing experiments continue to challenge our understanding.

1. Introduction

The more or less stable equilibrium structure of a foam has been understood in general terms since the 19th century [1]. It remains a subject of intense interest [2], but increasing attention is now being given to the dynamic side of the subject, having to do with liquid flow within the foam, or the shearing motion of the foam as a whole. This is not an easy subject, but the well-defined structure of the foam (figure 1) gives us hope of a detailed analysis of flow properties in due course, and some real insight into the factors that control its behaviour. A lot remains to be done and though a substantial book exists on the subject [3], those unfamiliar with the field should not be misled into believing that the physics of foams is now well founded in its entirety. Only relatively dry, static foams are well described by current models.

We begin our review of problems concerning the fluid dynamics of foam under drainage, that is, the transport of liquid through it.

2. Drainage

The liquid within a foam flows, with consequent changes in the local liquid fraction, in response to the local pressure gradient and gravity. Equilibrium is achieved when these two forces are

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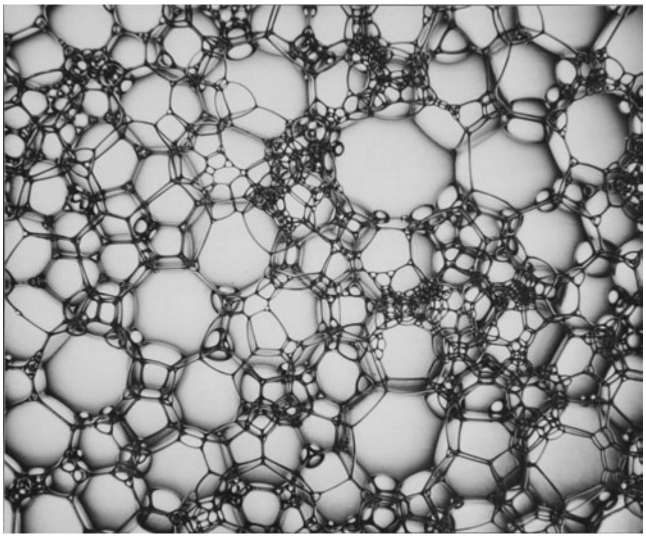

Figure 1. The structure of a liquid foam as seen by the photographer–artist Michael Boran.

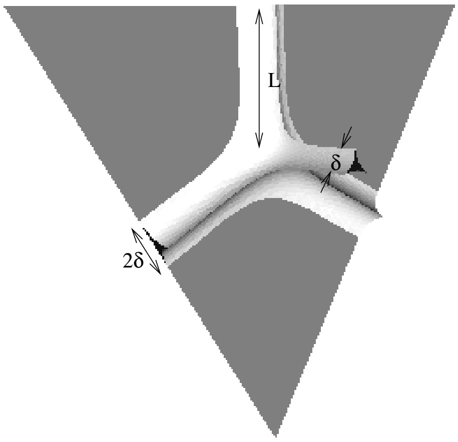

Figure 2. The flow of liquid through a foam occurs mainly in its Plateau borders. Here we show a junction of Plateau borders situated at a container surface. (The calculation was performed using the Surface Evolver [13].)

in balance. The pressure gradient in the liquid, if any, may be related to surface tension and hence is often described as 'capillarity'.

Elegant and successful theories have been developed for the case of a relatively dry foam, based on the assumption that fluid flow is mainly within the Plateau borders (figure 2), i.e. flow in the films is neglected [4–6]. A key question which has emerged is: what is the appropriate boundary condition at the boundaries of the Plateau border? Two extreme possibilities have been identified: a no-slip (or Poiseuille) condition and free slip (plug flow). In the latter case, since there is no resistance to flow in the Plateau borders, shear flow in the junctions comes into play as the dominant source of dissipation. Real systems may conform to one or the other (or indeed intermediate) behaviour.

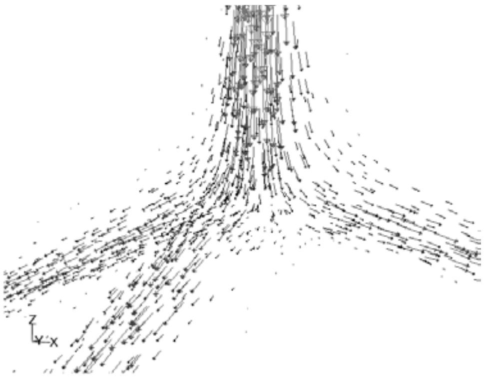

Figure 3. The velocity field for flow through a Plateau border junction computed with a no-slip boundary condition using the computational fluid dynamics package Fluent. The shape of the junction was obtained from a Surface Evolver calculation.

The two extreme cases are described by variants of the foam drainage equation. For one-dimensional analysis this can be written as

$$\frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial x}\left(\alpha^{k+1} - \frac{1}{2k+1}\frac{\partial \alpha^{k+1/2}}{\partial x}\right) = 0 \tag{1}$$

which describes the variation of the liquid fraction α with position x and time t (in dimensionless variables). The parameter k takes the values $k = 1/2$ for junction-dominated flow and $k = 1$ for Plateau-border-dominated flow [7]. Many analytical and numerical solutions have been reported for this equation for the two limiting values of k. Real systems do not conform precisely to either limit, so a generalized theory is needed.

Such equations are easily extended to describe three-dimensional drainage problems, which include cases of great practical interest such as foam flotation [8] and fractionation.

3. Local fluid flow

A more detailed understanding of drainage, particularly for higher liquid fractions, demands that we look more closely at the nature of local flow. In 1965 Robert Lemlich and his collaborators [9–11] had already begun to do this, and this particular investigator must be commended for his many insights.

An exciting recent development is the direct observation of flow profiles in Plateau borders using fluorescent latex spheres as markers [12]. However, since this type of experiment tends to be very difficult to execute and interpret, numerical modelling can provide complementary insight.

Figure 3 shows the velocity field for the flow created by an excess pressure applied to one border of a vertex. The boundary of the vertex is obtained from a static calculation in the Surface Evolver [13], and then the equations of slow viscous flow are solved numerically using the computational fluid dynamics package Fluent. In the case shown, we have applied a no-slip boundary condition. More details can be found in [14]. The next level of complexity is to consider how the flow deforms the boundaries that represent the elastic films.

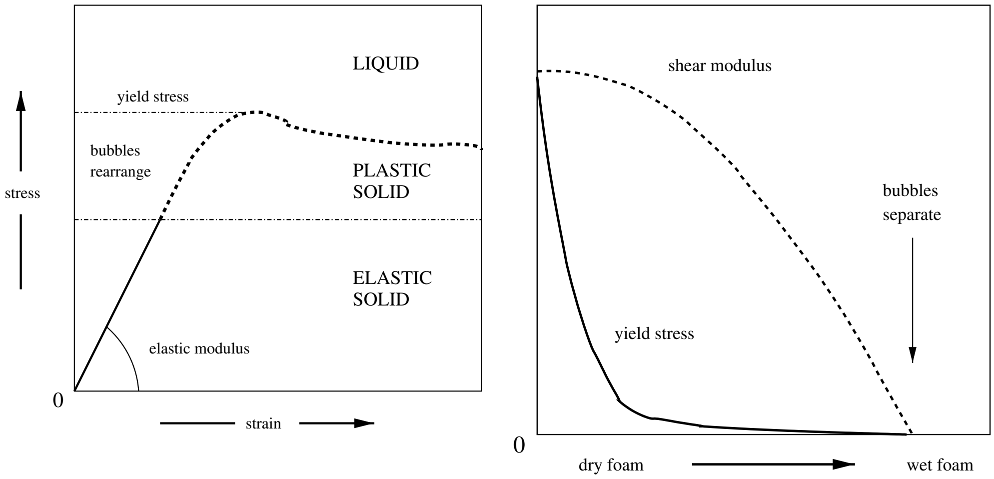

Figure 4. A sketch of the stress–strain relation for a liquid foam. Shear modulus and yield stress depend strongly on the liquid fraction of the foam. (The shear modulus is shown for the two-dimensional case; in three dimensions its variation is almost linear for the entire range of liquid fraction.)

4. Rheology

Simulations and experiments agree on the broad features of elastic properties of 2D and 3D foams [15, 16]. The shear modulus decreases smoothly with liquid fraction, vanishing in the wet limit. The same is true of the yield stress, and together these two functions of liquid fraction suffice to describe much of the behaviour of foams under stress (figure 4).

When the yield stress is exceeded, the foam will flow. In phenomenological Herschel–Bulkley descriptions of systems with a yield stress, the excess stress is resisted by a dissipative term proportional to some power of the strain rate. At the risk of oversimplification, one may focus on determining the value of this exponent. In another approach, the real and imaginary parts of the elastic response are examined as functions of frequency.

Recent experimental work focuses on the shearing of two-dimensional foams with the obvious advantage of visual observations [17, 18]. It appears that the outcome of such experiments, for example the observation of the formation of shear bands or the occurrence of avalanches of bubble rearrangements [19, 20], is highly dependent on experimental details. Three basic types of set-up are in use: one layer of bubbles may be contained between two glass plates, or it may sit on a pool of liquid, uncovered (Bragg raft) or covered by a contacting glass plate.

5. Convective instabilities

Attempts to use steady drainage as a means of creating uniformly wet foams have been frustrated by the occurrence of a convective instability [21, 22]. This is a circulation of the foam in which relatively wet foam travels downwards (either on one side or around the perimeter of a cylindrical column) and returns upwards with a lower liquid fraction (figure 5). For polydisperse foams, this circulation gives rise to a sorting of bubbles to create a vertical profile [23].

Recent measurements on monodisperse samples have shown that these two types of instability occur at liquid fractions of roughly 5–15%, depending on bubble size, as shown in figure 6 [24]. There is also considerable hysteresis.

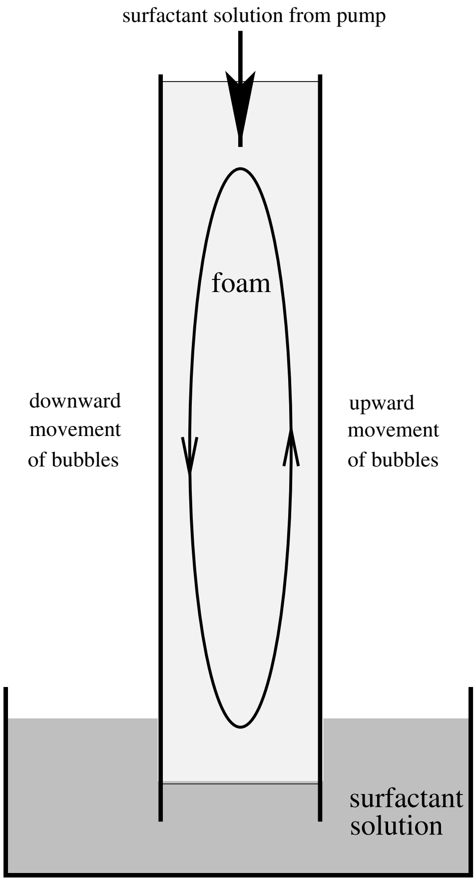

Figure 5. Forced drainage at large flow rates leads to the establishment of a convective bubble motion. The sketch illustrates one type of circulatory motion; the second type retains cylindrical symmetry.

The phenomenon has not been convincingly explained until now. Indeed, on the face of it, it is quite paradoxical. A slow convective roll can be established, in which there is a horizontal profile of liquid fraction. But, according to any naive theory using the standard drainage equation, this seems impossible, because liquid will diffuse horizontally to equilibrate pressures. We believe that the resolution of this paradox lies in the effect described in the next section.

6. Dilatancy

The term dilatancy was introduced by Osborne Reynolds in the 19th century, in the context of granular materials [25, 26]. It refers to the tendency of these materials, such as sand, to dilate (expand) under shear. We have shown that this effect is also significant in foams, particularly for intermediate liquid fractions [27]. This can supply the necessary rationale for the convective instability. The downward-moving part foam is continuously sheared, and dilatancy implies a higher liquid fraction for it (figure 7). This means that the liquid pressure remains the same throughout a horizontal section.

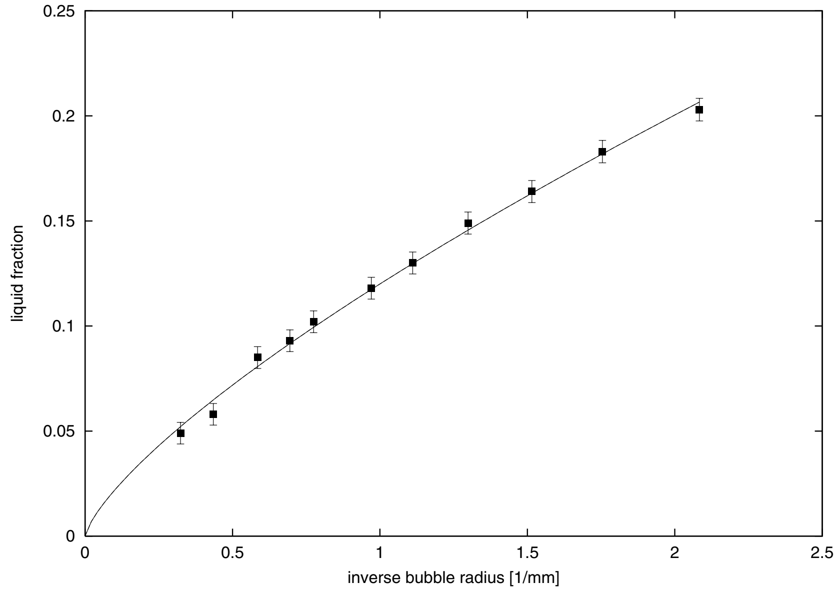

Figure 6. The dependence of the onset of convective motion on the bubble radius. The data shown are for a tube of diameter 2 cm; the length of the foam column is approximately 20 cm. The solid curve is a guide for the eye.

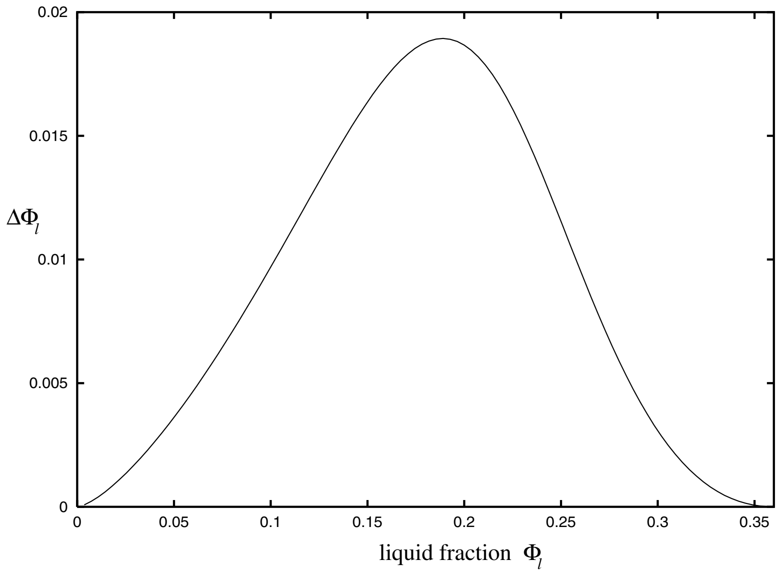

Figure 7. The estimated difference in liquid fraction between sheared and unsheared three-dimensional foam [27]. The estimate is based on experimentally obtained expressions for shear modulus, osmotic pressure and yield strain [16, 28, 29].

7. Magnetic foam

Magnetic foams made of ferrofluid have proved to be fascinating in 2D [30], and are now being explored in 3D. Particularly interesting is the creation of ordered structures in cylindrical tubes

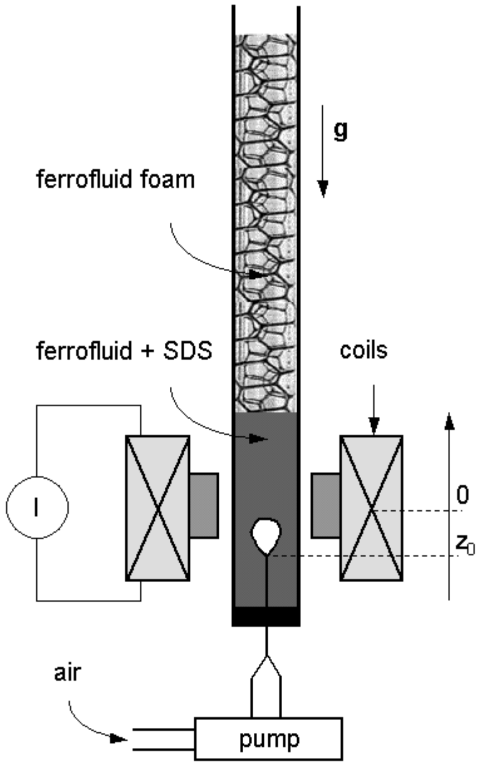

Figure 8. The experimental set-up for the production of ferrofluid foams. The bubble size depends on the magnitude and direction of the magnetic field gradient and thus can be tuned by varying the electric current through the coils. This allows for control of the foam structure in narrow tubes.

(see figure 8 for an illustration of the setup), since they may be manipulated in various ways by the application of a magnetic field [31]. Magnetic fields may also be used to control bubble size [32]. Furthermore, since ferrofluid foams are good conductors, foam structure can be detected by measuring local electric conductivity, leading to a direct measurement of bubble size. It is this combination of size control and structure manipulation and detection that makes ferrofluid foams good candidates for applications in the area of microfluidics.

8. Foam microfluidics

Many potentially useful tricks can be played with ordered foam structures (and presumably also emulsions) in cylindrical tubes or channels, of which an example is shown in figure 9. These processes are highly reproducible. Additional techniques may be brought to bear in the case of a ferrofluid foam, to control switching within the network [31]. All current experiments are on the scale of millimetres: it remains to be seen how easily such a methodology can be scaled down.

9. Conclusions

Foams have often seemed to present an ideal example of soft matter for both teaching and research. They illustrate clearly the case of a *yield stress material*, in a system which can

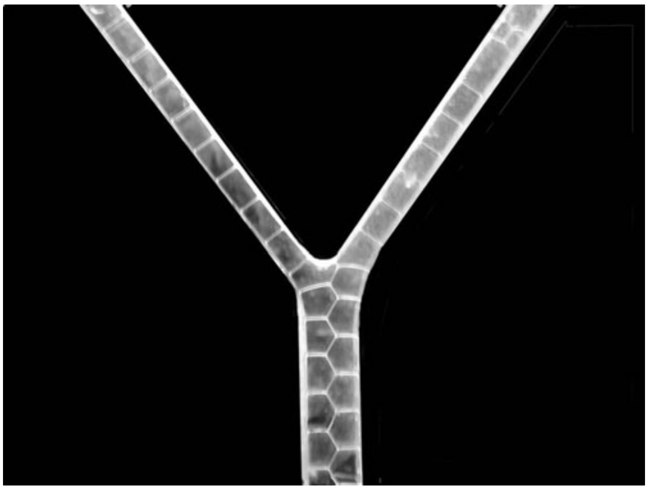

Figure 9. As the ordered structure moves up the pipe, it is split into two streams of bubbles, one moving to the left and the other to the right. The process can also be performed in reverse.

be probed easily and analysed in great detail. They continue to throw up fresh challenges which require the extension of existing theories, so the field remains remarkably lively. One must always be cautious in dealing with a material whose structure and properties are history dependent, but this can also be a source of fascination as one dimension of a rich subject.

Acknowledgments

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